Lip Dynamics in a Physical Model of the Trumpet

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ABSTRACT

A model of the trumpet has been studied in which the Navier-Stokes equations are used to calculate the air pressure and velocity, while the lips are described using the swinging lip model of Adachi and Sato. This approach allows us to determine the pressure throughout the lip region, and thus calculate the lip motion without the approximations necessary in previous modeling work.

1. INTRODUCTION

Models of musical instruments have become increasingly sophisticated in recent years, addressing questions that are difficult or impossible to tackle with experiments. Wind instruments such as the trumpet pose particular challenges, since modeling must include the dynamics of the air through the instrument as well as the motion of the player's lips. This paper describes an initial attempt to model the aeroacoustics of a brass instrument in a quantitatively accurate way using a direct application of the Navier-Stokes equations. While the instrument geometry we consider is admittedly simplified, our results should pave the way to the study of realistic geometries.

Essentially all previous modeling of brass instruments has assumed one-dimensional airflow and a Bernoulli-like approximation for the pressure near the lips (e.g., [1-4]). These approximations have been paired with various lip models, with the one devised by Adachi and Sato [2] appearing to be the simplest approach that captures the essential features of a "blown open" lip reed. While previous modeling of the trumpet has led to important insights, that work has significant limitations due to the approximate treatment of the pressure in the lip region. In this work we use a first-principles model of the airflow which allows us to compute the forces on the lips, and hence determine the lip motion, without many of the approximations made in previous work. We begin by describing our model and compare our results with previous work. We then present the first results for an important case that cannot be studied with previous modeling approaches - the behavior of asymmetric lips.

2. THE MODEL

Figure 1 shows our simplified model of the trumpet. It consists of a conical bore of length 5 cm with a diameter at the open end of 1.4 cm attached to rectangular lips that are able to swing in the direction of the cone axis in Fig. 1 and undergo stretching and compressive motion in the transverse direction. Our choice of such a small instrument reduces the computational time required for the aeroacoustic calculations.

Lip motion is described using the model of Adachi and Sato [2], according to which each lip experiences a Hooke's law restoring force when displaced from its equilibrium position, along with forces on the lip surfaces due to the air pressure.

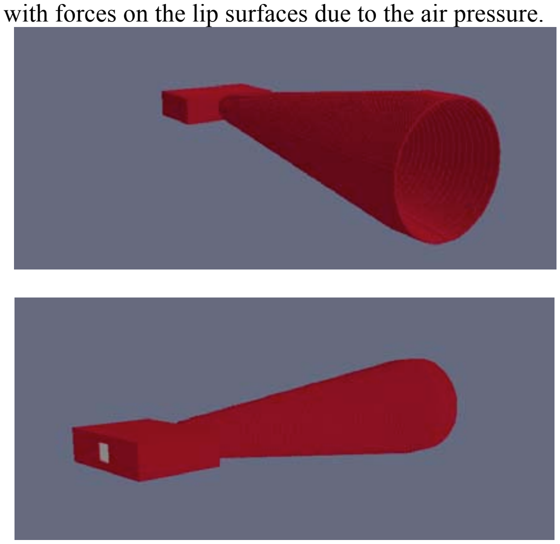

Figure 1. Model of the trumpet studied in this work. Top: Showing the conical bore open at the lower right. Bottom: The opening at the lower left leads inside to the mouth and lips, which are based on the description in [2].

There are a number of lip parameters in the model, including the mass m, stiffness (i.e., spring) constant k, and damping factor Q, along with the dimensions of the lips. For this study the lips were each 2.0 mm long (along the flow direction), 1.0 mm thick (in the flexible transverse direction denoted below by y), and 2.0 mm wide in the third dimension, and the nominal lip opening with no air flow was 0.6 mm along the y direction. The nominal cross sectional area of the lip channel was 1.2 mm^2 while that of the mouth (upstream from the lips) was 8.0 mm^2. The corresponding volumes were 2.4 mm^3 and 28 m^3. The lip mass for most of the calculations presented here was determined by assuming a lip density of 1000 kg/m^3, the approximate density of human tissue. The value of k then determined the natural frequency of each lip, and the damping was set by Q = 3, as suggested in [2]. Not surprisingly, the largest lip oscillations and largest sound pressure was found when the natural frequencies of the lips were close to one of the resonant frequencies of the bore, which was near 1 kHz. In such cases the lip oscillations could become very large and to maintain stability an extra restoring force was imposed when the lips touched (a similar approach was taken in [2]).

The air flow through the instrument was calculated using the compressible Navier-Stokes equations with the numerical approach described in [5]. The algorithm was a three-dimensional finite-difference time-domain calculation that yielded the air velocity and density on a grid with spacing 0.1 mm in the vicinity of the lips and inside the instrument. The results of studies of the recorder in [5] validated that our numerical method gives an accurate description of the flow at the length scale appropriate for the present work. The lips were treated using the immersed boundary method developed originally by Peskin [6] (see also [7,8] for a description of the method). With this method the lip edges are not limited to grid points, but are able to move continuously so the resolution of the lip motion is not limited by the grid spacing used in the Navier-Stokes calculation. The trumpet model was contained in a closed region with walls that reflect and absorb sound, as would the walls in a typical room [5]. The total number of grid points for the model considered here was 4.4×10^7.

In our algorithm [5] the instrument is "blown" by imposing a constant air velocity *u* that follows Poiseuille's law in a channel leading to the mouth cavity that then leads to the lips. That velocity was zero at $t = 0$ and increased linearly with time until a final value was reached at $t = 5$ ms; the blowing speed was then kept constant for the rest of the simulation. The blowing speeds required to produce lip oscillations and tones were somewhat larger than for a real trumpet, due to the small size of our model. For our chosen mouth geometry the pressure in the mouth was typically 10 kPa, a typical value for real playing, and the large mouth volume (relative to the lips) acted to somewhat buffer the flow as would be the case for a real player. The oscillations were stable for the duration of each simulation, with no hint of instabilities at long times. Steady state was generally reached after about 10 or 20 ms, and was then maintained for the duration of each simulation (typically 100 ms).

3. BEHAVIOR WITH SYMMETRIC LIPS

Some typical results for the sound pressure as a function of time with symmetric lips, i.e. with the two lips having the same dimensions, mass, stiffness constant, and other properties, are shown in Fig. 2. Corresponding results for the width of the opening between the lips are given in Fig. 3. In this case the lip oscillation was relatively large, and the lips sometimes closed for a short time during each oscillation cycle.

Figure 4 shows the motion of the two lips along *y*, the direction perpendicular to the net flow, where we plot the *y* coordinate of the center of each lip edge. Here $y = 0$ corresponds to the center of the channel between the lips; when the lips are in their equilibrium positions, each lip edge is 0.3 mm from the center of the channel. In Fig. 4 we expand the time scale slightly to show how the motions of the lips are synchronized as the lips move apart and then together (and nearly touch) each cycle. Figure 4 also shows that the amplitude of the motion is slightly larger for the upper lip in this case. We found that the lip oscillations were sometimes slightly asymmetric even though the lips themselves were nominally symmetric. There are several possible causes for this symmetry breaking. (1) While the lip parameters (mass, stiffness constant, and dimensions) were the same, the overall geometry was not perfectly symmetric as the instrument was placed slightly off center in the computational volume. (2) Two coupled oscillators that are degenerate can, if they interact, form two coupled modes with slightly different frequencies. (3) Such symmetry breaking could also be produced by small asymmetries in the numerical algorithm. The reason for the asymmetry found here is not clear and will be investigated in the future. We did observe that the best symmetry was generally found when the lip oscillations were large and with the lip frequency close to a resonant frequency of the bore.

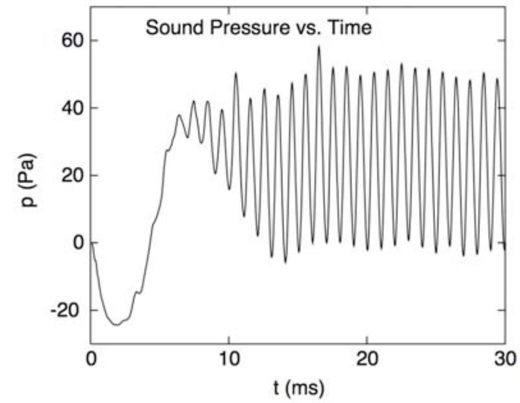

Figure 2. Sound pressure as a function of time at a location outside the trumpet and off axis relative to the bore. Lip parameters were $m = 4 \times 10^{-6}$ kg, $k = 125$ N/m, and $Q = 3$. The blowing speed at the center of the channel leading to the mouth was 200 m/s.

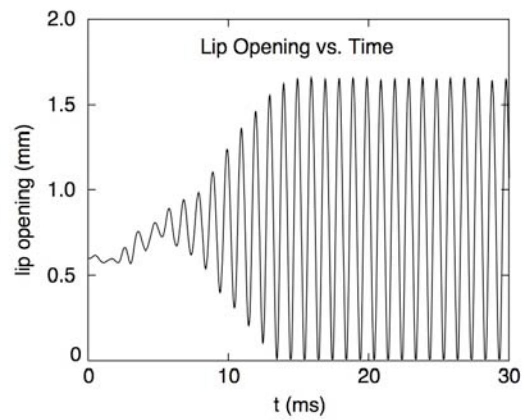

Figure 3. Variation of the width of the lip channel with time, for the simulation in Fig. 2.

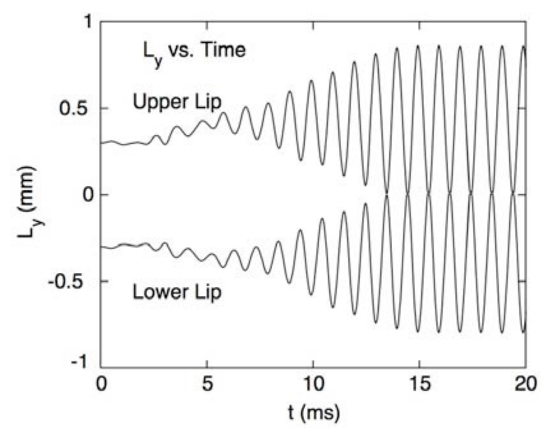

Figure 4. Motion of the upper and lower lips along the *y* direction (perpendicular to the net flow) for the simulation in Figs. 2 and 3.

Figure 5 shows the lip motion in a different way, plotting the lip displacement along y (the direction perpendicular to the net flow) versus the displacement along x (the direction parallel to the net flow) after the steady state oscillation was reached. While there is no indication of time in Fig. 5, the oscillations of the two lips were very close to being mirror images with respect to the $y = 0$ axis (Fig. 4). The lips thus nearly touch when $x \sim +0.1$ mm. The equilibrium lip positions are at $x = 0$ so the lips spend most of each cycle blown open, as expected, although they do swing back a small distance counter to the direction of net flow.

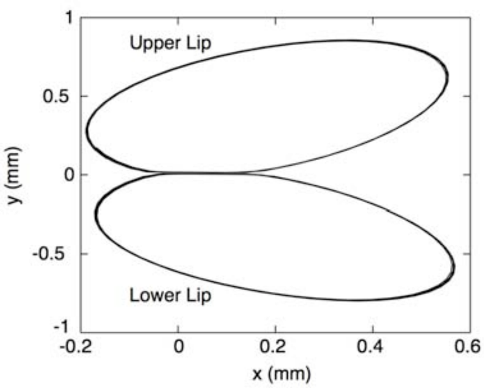

Figure 5. Motion of the center of the edge of each lip during successive oscillation cycles. Motion along y corresponds to squeezing and stretching of the lips, while motion along x corresponds to swinging motion parallel to the direction of net flow. Compare with Fig. 5 in [2].

4. THE CASE OF ASYMMETRIC LIPS

The results in Figs. 2-5 were all obtained with lips that were symmetric, that is, with the same mass, stiffness, and dimensions, and we found that the lip motion was approximately symmetric. To the best of our knowledge, all previous trumpet modeling has assumed symmetric motion of the lips. However, the lips of a real player are not symmetric, and our use of the Navier-Stokes equations to treat the air motion and pressure in three dimensions within the instrument allows us to readily consider the case of non-symmetric lips. In general one would expect both the mass and stiffness to be different for the upper and lower lips of a real player, leading at least to different oscillation amplitudes (as is indeed found experimental studies of the lip motion for real players of brass instruments [9-13]). For simplicity we will in this section consider just two cases: (1) lips with the same masses and different stiffness constants, and (2) lips with the different masses and the same stiffness constants. In both cases the two lips will be taken to have the same dimensions as in model considered above, and all other dimensions of the model were the same.

We first consider case (1), with lips having the same mass as in Figs. 2-5 but different stiffness factors k. The ratio of the stiffness factors was chosen to be 2, so since the natural frequency of a lip varies as $k^{1/2}$, the natural frequencies of the two lips differ by about 40% in this case. Figure 6 shows results for the width of the lip opening as a function of time; we find an oscillation at a single frequency that is approximately the same as the resonant frequency of the bore and the same as seen in Figs. 2-5, suggesting that coupling through the resonant motion of the surrounding air has caused the lips to move in synchrony. This is confirmed in Fig. 7, which shows the lip motion along y; indeed, the results in Figs. 6 and 7 are quantitatively nearly identical to the case with symmetric lips in Figs. 4 and 5. Note also that the two lips again move in phase; that is, they reach their maxima (of the lower lip) and minima (of the upper lip) at the same time. This synchrony and phase locking of the motion of the two lips is maintained for very long times (100 ms or more) which is easily long enough to reveal a 40% difference in the frequencies of the two lips if that were the case. Our results show that the Q factor is low enough that the lips are able to lock to the frequency of the conical bore.

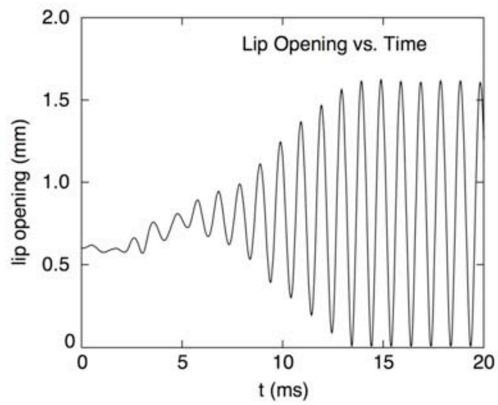

Figure 6. Variation of the width of the lip opening with time, for the case of lips with different stiffness factors: $m = 4\text{x}10^{-6}$ kg, $k_{\text{lower}} = 125$ N/m, $k_{\text{upper}} = 60$ N/m.

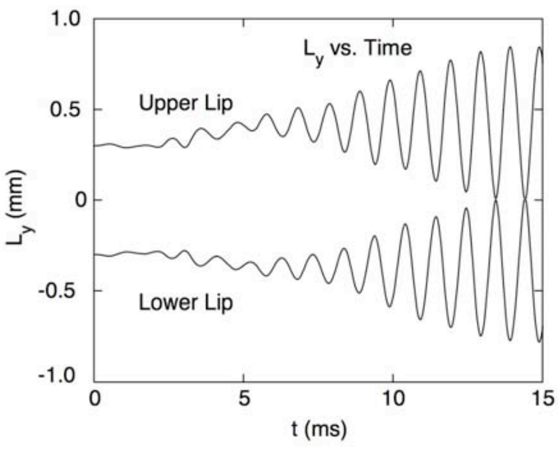

Figure 7. Lip motion along y for lips with different stiffness factors as given in the caption for Fig. 6.

Figures 8 and 9 show results for lips with different masses and the same stiffness. The ratio m(upper)/m(lower) = 2, so the natural frequencies of the two lips differed by about 40%, as for the case considered in Figs. 6 and 7. In Figs. 8 and 9 we use the same scales as in Figs. 6 and 7 to emphasize that the results are now very different from the behavior found with symmetric lips. The resulting lip vibration amplitude is much smaller than found with either symmetric lips or with lips of the same mass but different stiffness. We also see from Fig. 9 that the two lips do not even lock to a common frequency.

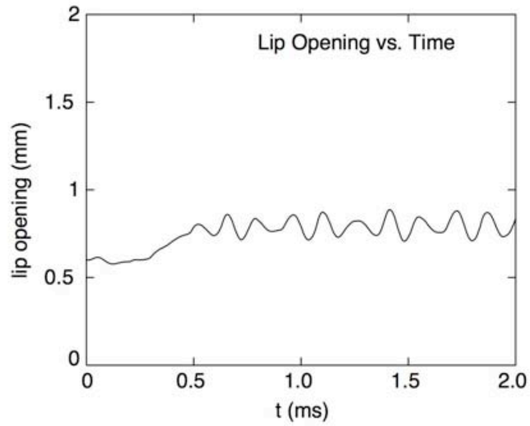

Figure 8. Variation of the width of the lip opening with time, for the case of lips with different masses and the same stiffness: $m_{\text{lower}} = 4\text{x}10^{-6}$ kg, $m_{\text{upper}} = 8\text{x}10^{-6}$ kg, $k = 125$ N/m.

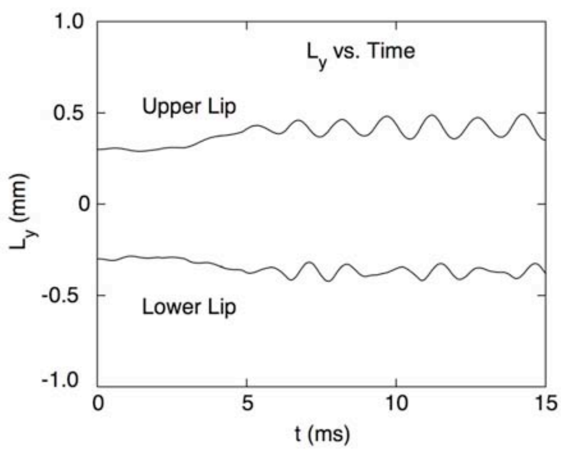

Figure 9. Lip motion along y for lips with different masses as given in the caption for Fig. 8.

5. CONCLUSIONS

This paper presents first results for a trumpet model in which the air is treated using the compressible Navier-Stokes equations in three dimensions. For the case of symmetric lips we find results that are similar in many respects to that found previously in modeling that treated the air flow in a more approximate way. We have also obtained results for the more realistic case of asymmetric lips. For lips in which the asymmetry is due to different stiffness factors, the lips move in synchrony and in phase at a single frequency producing what should be a good musical tone. When a roughly comparable asymmetry is present with lips of differing mass, the lips move slightly out of phase resulting in a much reduced sound amplitude.

Our results suggest many avenues for future work. The behavior for asymmetric lips with masses that are more nearly equal will be studied to determine if and when synchrony occurs, and the case of lips which differ in both mass and stiffness must be explored. It will also be interesting to extend our work to include more complicated lip models, like those used in studies of phonation [14]. In addition to the lip motion, our modeling approach yields the spatial and temporal variations of the pressure and air velocity throughout the mouthpiece-lip region, which will be compared with experiments in future work. It is also possible to make movies of the lip motion and flow velocity, and these will be presented elsewhere.

6. ACKNOWLEDGEMENTS

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